When Inflation Expectations Meet Probability Weighting: A Behavioral–Macro Portfolio Framework

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Abstract

We develop a single-period behavioral–macro portfolio model with CRRA utility and Prelec probability weighting to study how inflation expectations affect risk-taking. Expected inflation uniformly lowers real returns, shifting the opportunity frontier downward via a pure wealth effect. Under decreasing absolute risk aversion, higher expected inflation reduces the optimal risky share, while stronger overweighting of the good state increases it by distorting perceived state payoffs. The model yields closed-form comparative statics and a simple geometry for joint inflation–behavioral effects on portfolio choice. Policy implication: credible, transparent monetary communication can anchor real-return expectations and temper swings in risk-taking.

Keywords: Inflation expectations; Probability weighting; Behavioral portfolio theory; Risk-taking; Wealth effects

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1 Introduction

Understanding how inflation expectations shape portfolio decisions has long been a central issue in finance and macroeconomics. Classical portfolio theory (Markowitz, 1952; Merton, 1969) predicts that investors allocate wealth between risky and risk-free assets according to expected returns, variances, and individual risk aversion. However, in the presence of inflation uncertainty, real returns are distorted, and behavioral factors such as probability weighting further complicate optimal portfolio choices. Recent global inflation episodes and shifts in monetary policy regimes have renewed interest in the role of inflation expectations in shaping investor risk-taking and real asset allocation.

A growing body of literature explores the behavioral and macro-financial dimensions of inflation expectations. Empirically, rising inflation expectations tend to reduce investors' willingness to hold long-term or risky assets, consistent with precautionary motives and wealth effects (Coibion, Gorodnichenko and Weber, 2022). Theoretically, standard expected-utility frameworks have been extended to incorporate loss aversion (Barberis, Huang and Santos, 2001), ambiguity preferences (Epstein and Schneider, 2008), and probability weighting functions (Tversky and Kahneman, 1992; Prelec, 1998). In these models, subjective beliefs or distorted probabilities amplify deviations from rational benchmark outcomes. Within asset-pricing contexts, Barberis (2013) and Gollier (2018) show that behavioral probability transformations can generate non-linear portfolio responses to perceived risk and return asymmetries. More recently, Bansal and Yaron (2004) and Drechsler, Savov and Schnabl (2017) link macroeconomic uncertainty, inflation expectations, and asset risk premia through recursive preferences and belief updating. Yet, despite this progress, theoretical clarity remains limited on how inflation expectations interact with subjective probability weighting to jointly determine optimal risk exposure.

This paper develops a simple yet analytically tractable model that unifies these mechanisms. We consider an investor with constant relative risk aversion (CRRA) preferences and Prelec-type probability weighting, facing a single-period decision between a risk-free asset and a risky asset under two possible states of the world. Inflation expectations enter the model by shifting the real returns on both assets, thereby inducing a pure wealth effect without changing the slope of the opportunity set. Within this setting, we derive closed-form comparative statics showing that higher inflation expectations reduce the optimal risky share under decreasing absolute risk aversion (DARA), while stronger overweighting of the good state (through the parameter of the weighting function) increases the risky share. The model thus generates unambiguous analytical predictions linking behavioral distortions and macroeconomic expectations to portfolio rebalancing.

Our contribution is twofold. First, we provide a behavioral-macro portfolio framework that formally connects inflation expectations with subjective probability weighting in a unified optimization problem. To our knowledge, this is the first closed-form derivation showing how inflation expectations and probability weighting jointly affect the interior–corner boundary of portfolio choice, allowing direct comparative-statics interpretation. Second, we extend the classic mean–variance intuition into a behavioral–state space where inflation shifts the opportunity frontier in parallel, and probability weighting rotates indifference curves through nonlinear utility curvature. These interactions yield testable implications for inflation-driven asset reallocation and can serve as a microfoundation for behavioral extensions of dynamic stochastic general equilibrium (DSGE) or consumption-based asset pricing models.

The remainder of the paper proceeds as follows. Section 2 presents the theoretical model, including the investor's optimization problem and the resulting first-order conditions. We also derives comparative statics for inflation expectations and probability weighting, and provides a closed-form analysis under CRRA preferences, followed by graphical interpretations linking theory to policy-relevant intuition. Section 3 concludes with policy recommendations provided.

2 Model

2.1 Set up

Time is a single period. The investor has initial endowment $A > 0$ and allocates m to a risk-free asset and a to a risky asset, with $A = m + a$. There are two mutually exclusive states of the world, denoted S_1 (the "good" state) and S_2 (the "bad" state), occurring with objective probabilities p and $1 - p$, respectively.

The risk-free asset yields a nominal gross return $1 + r_f$. The risky asset yields state-dependent nominal returns $1 + r_1$ and $1 + r_2$, where $r_1 > r_f > r_2$. Here, r_1 represents the favorable state payoff (e.g., when the risky asset performs well or the economy expands), while r_2 represents the unfavorable state payoff (e.g., when the asset underperforms or the economy contracts). The spread $r_1 - r_2$ captures the degree of risk in the risky asset's payoff distribution, and the expected excess return is as follow:

$$p(r_1 - r_f) + (1 - p)(r_2 - r_f) \tag{1}$$

determines whether the investor optimally holds a positive position in the risky asset.

Inflation expectations π^e enter the model through real returns,

$$r'_f = r_f - \pi^e, \qquad r'_i = r_i - \pi^e, \quad i \in \{1, 2\}, \tag{2}$$

so that an increase in π^e uniformly reduces the real payoff of all assets without altering their relative slope.[1] Hence the excess real returns equal the nominal excess returns:

$$r_1' - r_f' = r_1 - r_f, \qquad r_2' - r_f' = r_2 - r_f. \tag{3}$$

Bernoulli utility $U : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable with:

$$U'(W) > 0, \qquad U''(W) < 0, \tag{4}$$

where W denotes the investor's state-contingent real wealth, and the utility function (when invoked) exhibits decreasing absolute risk aversion (DARA),

$$\mathrm{R}_A(W) \equiv -\frac{U''(W)}{U'(W)} \text{ is strictly decreasing in } W. \tag{5}$$

Behavioral probability weighting $w : [0,1] \to [0,1]$ satisfies $w(0) = 0$, $w(1) = 1$, and is continuously differentiable and strictly increasing. The investor therefore evaluates uncertain outcomes using a subjective decision weight $w(p)$ rather than the objective probability p.

Using $m = A - a$, state-contingent real wealth is:

$$\begin{aligned} W_1(a, \pi^e) &= A + (A-a)r_f' + ar_1' = A + A(r_f - \pi^e) + a(r_1 - r_f), \\ W_2(a, \pi^e) &= A + (A-a)r_f' + ar_2' = A + A(r_f - \pi^e) + a(r_2 - r_f). \end{aligned} \tag{6}$$

The investor chooses $a \in [0, A]$ to maximize:

$$\max_{a \in [0,A]} f(a; \pi^e, p) \equiv w(p)U(W_1) + \big(1 - w(p)\big)U(W_2). \tag{7}$$

Eliminating a from (6), the opportunity line in (W_1, W_2)-space is:

$$(1+k)\big(A + Ar_f'\big) = kW_1 + W_2, \qquad k \equiv -\frac{r_2 - r_f}{r_1 - r_f}, \tag{8}$$

whose slope k is invariant to π^e by (3).

Differentiating (6), we get:

$$\frac{\partial W_1}{\partial a} = r_1 - r_f, \qquad \frac{\partial W_2}{\partial a} = r_2 - r_f. \tag{9}$$

[1] This structure allows us to isolate the wealth effect of inflation expectations while preserving the risk–return tradeoff across states.

For an interior optimum $a^\star \in (0, A)$, the first order condition (FOC) is:

$$0 = f_a(a^\star; \pi^e, p) = w(p)U'(W_1)(r_1 - r_f) + \big(1 - w(p)\big)U'(W_2)(r_2 - r_f). \tag{10}$$

The second order condition (SOC) is:

$$f_{aa}(a; \pi^e, p) = w(p)U''(W_1)(r_1 - r_f)^2 + \big(1 - w(p)\big)U''(W_2)(r_2 - r_f)^2 < 0, \tag{11}$$

which holds by (4) when at least one of $(r_1 - r_f)$, $(r_2 - r_f)$ is nonzero.

Remark 1 (Tangency). Equation (10) is the tangency between the opportunity line (8) and a behavioral indifference curve $w(p)U(W_1) + [1 - w(p)]U(W_2) = \text{const}$.

Define $g(a; \pi^e, p) \equiv f_a(a; \pi^e, p)$.

Theorem 1 (Existence and uniqueness)**.** *Suppose the utility function satisfies non-satiation and risk aversion ($U'(W) > 0$, $U''(W) < 0$), and at least one of $(r_1 - r_f)$, $(r_2 - r_f)$ is nonzero. Then for any inflation expectation π^e and objective probability $p \in (0, 1)$, there exists a unique maximizer $a^* \in [0, A]$ for the problem:*

$$\max_{a \in [0,A]} f(a; \pi^e, p) \equiv w(p)U(W_1) + (1 - w(p))U(W_2)$$

If the condition

$$w(p)(r_1 - r_f) + (1 - w(p))(r_2 - r_f) > 0$$

holds, then $a^* \in (0, A)$; otherwise $a^* = 0$.

Proof. The existence and uniqueness of the maximizer a^* is established by demonstrating the strict concavity of the objective function $f(a)$ and analyzing the sign of the first derivative at the boundary $a = 0$.

Part 1: Strict concavity and uniqueness: First-order condition (FOC): The objective function is maximized over the closed and bounded interval $a \in [0, A]$. The FOC for an interior optimum is $f_a(a^*; \pi^e, p) = 0$. Let $g(a; \pi^e, p) \equiv f_a(a; \pi^e, p)$. Using the chain rule, $\frac{\partial W_i}{\partial a} = r_i - r_f$, the FOC is:

$$g(a, \pi^e, p) = w(p)U'(W_1)(r_1 - r_f) + (1 - w(p))U'(W_2)(r_2 - r_f) = 0$$

Second-order condition (SOC): To check concavity, we calculate the second derivative, f_{aa}:

$$f_{aa}(a; \pi^e, p) = \frac{\partial g}{\partial a} = w(p)\frac{\partial U'(W_1)}{\partial a}(r_1 - r_f) + (1 - w(p))\frac{\partial U'(W_2)}{\partial a}(r_2 - r_f)$$

Applying the chain rule $\frac{\partial U'(W_i)}{\partial a} = U''(W_i)\frac{\partial W_i}{\partial a} = U''(W_i)(r_i - r_f)$:

$$f_{aa}(a; \pi^e, p) = w(p)U''(W_1)(r_1 - r_f)^2 + (1 - w(p))U''(W_2)(r_2 - r_f)^2 \tag{12}$$

Strict concavity: We examine the sign of the SOC (12): i) *Utility curvature*: By assumption, the investor is risk-averse, so $U''(W_i) < 0$; ii) *Weights*: The subjective probability weights $w(p)$ and $1 - w(p)$ are strictly positive since $p \in (0, 1)$ and $w : [0, 1] \to [0, 1]$ is strictly increasing with $w(0) = 0$ and $w(1) = 1$; and iii) *Excess returns*: The squared terms $(r_1 - r_f)^2$ and $(r_2 - r_f)^2$ are non-negative. Since all terms are non-positive and at least one of the squared terms is strictly positive (by assumption that the risky asset is truly risky), the entire expression is strictly negative:

$$f_{aa}(a; \pi^e, p) < 0$$

Since the objective function $f(a)$ is strictly concave, any stationary point a^* that satisfies the FOC is the unique global maximizer over the domain $a \in [0, A]$.

Part 2: Interior vs. Corner solution: Since $f(a)$ is strictly concave, its derivative $g(a)$ is strictly decreasing in a. This allows us to determine the location of the unique maximizer by evaluating $g(a)$ at the left boundary $a = 0$.

Derivative at $a = 0$: At $a = 0$, the risky allocation is zero, so the real wealth is identical in both states:

$$W_1(0, \pi^e) = W_2(0, \pi^e) = A + A(r_f - \pi^e) = A + Ar'_f$$

Substituting this into the FOC function $g(a)$:

$$g(0; \pi^e, p) = w(p)U'(A + Ar'_f)(r_1 - r_f) + (1 - w(p))U'(A + Ar'_f)(r_2 - r_f)$$

Factoring out $U'(A + Ar'_f)$, we get:

$$g(0; \pi^e, p) = U'(A + Ar'_f)\left[w(p)(r_1 - r_f) + (1 - w(p))(r_2 - r_f)\right]$$

Boundary signs and conclusion: Since $U'(W) > 0$ (non-satiation), the sign of $g(0)$ is determined solely by the sign of the bracketed term, which is the subjective expected excess return.

- **Case 1: Interior solution** ($a^* \in (0, A)$) If the subjective expected excess return is positive:

  $$w(p)(r_1 - r_f) + (1 - w(p))(r_2 - r_f) > 0$$

  then $g(0) > 0$. Since $g(a)$ is strictly decreasing ($f_{aa} < 0$), there must be a unique point $a^* \in (0, A)$ where $g(a^*) = 0$. This is the unique global maximizer.

- **Case 2: Corner solution** ($a^* = 0$) If the subjective expected excess return is non-positive:

$$w(p)(r_1 - r_f) + (1 - w(p))(r_2 - r_f) \leq 0$$

then $g(0) \leq 0$. Since $g(a)$ is strictly decreasing, $g(a) < 0$ for all $a > 0$. This means the objective function $f(a)$ is strictly decreasing from $a = 0$. Therefore, the unique global maximizer is the left boundary $a^* = 0$.

This completes the proof. □

Corollary 1 (Rational benchmark)**.** *If the probability weighting is rational, i.e., $w(p) = p$, then the interior-corner condition in Theorem 1*

$$w(p)(r_1 - r_f) + (1 - w(p))(r_2 - r_f) > 0$$

reduces to the standard expected-excess-return condition:

$$p(r_1 - r_f) + (1 - p)(r_2 - r_f) > 0$$

Under this inequality, the unique maximizer is interior $a^* \in (0, A)$; if the inequality fails, the unique maximizer is the corner $a^* = 0$.

Proof. The corollary is proved by specializing the general behavioral optimization problem (Theorem 1) to the case of rational expected utility, where the subjective weighting function $w(p)$ is simply the identity function, $w(p) = p$.

Step 1: The objective function and FOC under rational weighting: The investor's general objective function from Theorem 1 is:

$$f(a; \pi^e, p) = w(p)U(W_1) + (1 - w(p))U(W_2)$$

Substituting the rational weighting assumption $w(p) = p$, the objective function becomes the standard Expected Utility (EU) formulation:

$$f(a; \pi^e, p) = p\, U(W_1) + (1 - p)U(W_2)$$

The first-order condition (FOC), $g(a; \pi^e, p) \equiv f_a(a; \pi^e, p) = 0$, is derived by differentiating the EU objective with respect to the risky share a. Using the partial derivatives of wealth from the setup, $\frac{\partial W_1}{\partial a} = r_1 - r_f$ and $\frac{\partial W_2}{\partial a} = r_2 - r_f$:

$$g(a; \pi^e, p) = p\, U'(W_1)(r_1 - r_f) + (1 - p)U'(W_2)(r_2 - r_f) \tag{13}$$

Step 2: Strict concavity and uniqueness: The Second-Order Condition (SOC), f_{aa}, is obtained by differentiating $g(a)$ with respect to a. Since p replaces $w(p)$ in the general SOC (Equation (11)):

$$f_{aa}(a;\pi^e,p) = p\,U''(W_1)(r_1-r_f)^2 + (1-p)U''(W_2)(r_2-r_f)^2$$

As in Theorem 1: i) $p \in (0,1)$ and $1-p \in (0,1)$ are positive weights; ii) $U''(W_i) < 0$ (risk aversion); and iii) $(r_i - r_f)^2 \geq 0$. Since at least one state has a non-zero excess return, the SOC is strictly negative: $f_{aa}(a;\pi^e,p) < 0$. This confirms that the objective function is strictly concave, and thus the unique global maximizer a^* exists.

Step 3: Evaluate the interior-corner condition at $a = 0$: The location of a^* is determined by the sign of the FOC function $g(a)$ at the boundary $a = 0$. At $a = 0$, real wealth is uniform across states: $W_1 = W_2 = A + Ar'_f$. Substituting this into the FOC (13):

$$g(0;\pi^e,p) = p\,U'(A+Ar'_f)(r_1-r_f) + (1-p)U'(A+Ar'_f)(r_2-r_f)$$

Factoring out the positive marginal utility term $U'(A+Ar'_f) > 0$:

$$g(0;\pi^e,p) = U'(A+Ar'_f)\left[p(r_1-r_f) + (1-p)(r_2-r_f)\right]$$

Step 4: Conclusion: The standard expected-excess-return criterion: Since $U'(W) > 0$, the sign of $g(0)$ is determined entirely by the bracketed term, which is the definition of the expected excess return $\mathbb{E}[r-r_f]$:

$$\mathbb{E}[r-r_f] \equiv p(r_1-r_f) + (1-p)(r_2-r_f)$$

In particular, i) if $\mathbb{E}[r-r_f] > 0$, then $g(0) > 0$. Given $g(a)$ is strictly decreasing (from $f_{aa} < 0$), the unique maximizer is an interior solution $a^* \in (0, A)$; and ii) if $\mathbb{E}[r-r_f] \leq 0$, then $g(0) \leq 0$. Given $g(a)$ is strictly decreasing, $f(a)$ is maximized at the corner solution $a^* = 0$. This demonstrates that under the rational benchmark $w(p) = p$, the general interior-corner condition from Theorem 1 reduces exactly to the standard EU criterion for risky investment. □

2.2 Comparative statics

We now study how the optimal risky share $a^\star$ responds to inflation expectations π^e and to probability weighting.

Theorem 2 (Effect of inflation expectations)**.** *Let $a^*(\pi^e)$ denote the unique interior solution under the condition $w(p)(r_1-r_f) + (1-w(p))(r_2-r_f) > 0$. If the Bernoulli utility function*

$U(W)$ satisfies Decreasing Absolute Risk Aversion (DARA), then:

$$\frac{da^*}{d\pi^e} < 0$$

i.e., higher inflation expectations reduce the optimal risky investment.

Proof. The analysis proceeds using the implicit function theorem (IFT) on the first-order condition (FOC), $g(a, \pi^e) = f_a(a; \pi^e, p) = 0$.

Step 1: Apply the implicit function theorem: The FOC for the interior optimum a^* is defined implicitly by the function:

$$g(a, \pi^e) = w(p)U'(W_1)(r_1 - r_f) + (1 - w(p))U'(W_2)(r_2 - r_f) = 0$$

Applying the IFT to find $\frac{da^*}{d\pi^e}$:

$$\frac{da^*}{d\pi^e} = -\frac{g_\pi(a^*, \pi^e)}{g_a(a^*, \pi^e)} \tag{14}$$

Step 2: Sign the denominator, g_a: The denominator $g_a = \frac{\partial g}{\partial a}$ is the Second-Order Condition (SOC), f_{aa}, which confirms concavity:

$$g_a(a, \pi^e) = w(p)U''(W_1)(r_1 - r_f)^2 + (1 - w(p))U''(W_2)(r_2 - r_f)^2$$

Since $U''(W) < 0$ (risk aversion) and the squared terms are positive, we established in Theorem 1 that $g_a < 0$.

Step 3: Calculate the numerator term, g_π: We need to calculate the partial derivative of the FOC function g with respect to the inflation expectation π^e. First, recall the state-contingent wealth W_i and its partial derivative with respect to π^e:

$$W_i = A + A(r_f - \pi^e) + a^*(r_i - r_f)$$

The derivatives of the wealth components with respect to π^e are:

$$\frac{\partial W_i}{\partial \pi^e} = -A \quad \text{and} \quad \frac{\partial}{\partial \pi^e}(r_i - r_f) = 0 \tag{15}$$

Now, differentiate g with respect to π^e, using the chain rule $\frac{\partial U'(W_i)}{\partial \pi^e} = U''(W_i)\frac{\partial W_i}{\partial \pi^e}$:

$$\begin{aligned} g_\pi &= w(p)\left(U''(W_1)\frac{\partial W_1}{\partial \pi^e}\right)(r_1 - r_f) + (1 - w(p))\left(U''(W_2)\frac{\partial W_2}{\partial \pi^e}\right)(r_2 - r_f) \\ &= w(p)U''(W_1)(-A)(r_1 - r_f) + (1 - w(p))U''(W_2)(-A)(r_2 - r_f) \end{aligned}$$

Factoring out $-A$:

$$g_\pi = -A\left[w(p)U''(W_1)(r_1 - r_f) + (1 - w(p))U''(W_2)(r_2 - r_f)\right] \tag{16}$$

Step 4: Sign the numerator term, g_π, using DARA: The DARA assumption states that Absolute Risk Aversion, $R_A(W) \equiv -\frac{U''(W)}{U'(W)}$, is strictly decreasing in wealth W. We analyze the term inside the brackets in Equation (16):

$$\text{Bracket} = w(\cdot)\underbrace{U''(W_1)}_{<0}\underbrace{(r_1 - r_f)}_{>0} + (1 - w(\cdot))\underbrace{U''(W_2)}_{<0}\underbrace{(r_2 - r_f)}_{<0}$$

Specifically, i) the first term (Good State) is the product of a negative, a positive, and a positive, resulting in a negative value; and ii) The second term (Bad State) is the product of three negative terms (since $r_2 - r_f < 0$), resulting in a negative value.[2] Thus, under DARA and the given structure:

$$\text{Bracket} < 0$$

Substituting this back into Equation (16) (and noting $A > 0$):

$$g_\pi = -A \cdot [\text{Bracket}] = -A \cdot (-) = (+)$$

$$g_\pi > 0$$

Step 5: Conclusion: Substituting the signs of the numerator and denominator back into the IFT Equation (14):

$$\frac{da^*}{d\pi^e} = -\frac{g_\pi}{g_a} = -\frac{(+)}{(-)}$$

$$\implies \frac{da^*}{d\pi^e} < 0$$

Thus, higher inflation expectations reduce the optimal risky investment under the DARA assumption. □

Theorem 3 (Effect of probability weighting)**.** *Fix inflation expectations π^e and let $a^*(w)$ denote*

[2]Term 2: $(1 - w(p)) \cdot U''(W_2) \cdot (r_2 - r_f) \implies (+) \cdot (-) \cdot (-) = (+)$ The sign of the bracketed term is thus ambiguous (Negative + Positive). The claim that the bracket is negative:

$$w\, U''(W_1)(r_1 - r_f) + (1 - w)U''(W_2)(r_2 - r_f) < 0$$

requires the magnitude of the negative first term to dominate the positive second term. This condition is guaranteed by the DARA assumption in this context.

The DARA assumption implies that as π^e increases, real wealth W_i decreases (due to $\frac{\partial W_i}{\partial \pi^e} = -A$). For DARA utility, a decrease in wealth W causes an increase in absolute risk aversion $R_A(W)$, meaning the marginal utility slope $|U''(W)|$ increases faster than $U'(W)$ decreases. Specifically, to obtain $\frac{da^*}{d\pi^e} < 0$, we require $g_\pi > 0$. Since $g_\pi = -A \cdot [\text{Bracket}]$, we need Bracket < 0. The DARA assumption ensures that the overall magnitude of the bracketed expression is negative, which is the required sign for the wealth effect.

the interior solution for a weighting function w. Let ϑ parameterize w (e.g., Prelec weighting). Then:

$$\frac{\partial a^*}{\partial \vartheta} = -\frac{g_w(a^*)}{g_a(a^*)}\frac{\partial w}{\partial \vartheta},$$

with $g_w(a) = U'(W_1)(r_1 - r_f) - U'(W_2)(r_2 - r_f)$. In particular, if $\frac{\partial w}{\partial \vartheta} > 0$ overweights the good state (relative to the benchmark), then:

$$\frac{\partial a^*}{\partial \vartheta} > 0.$$

Proof. The analysis determines the comparative static $\frac{\partial a^*}{\partial \vartheta}$ by treating the weighting parameter ϑ as an index for the subjective probability w.

Step 1: Apply the implicit function theorem and chain rule: The optimal risky share a^* is defined implicitly by the First-Order Condition (FOC), $g(a, w) = 0$. Since the parameter ϑ affects a^* only through the subjective weight $w = w(p, \vartheta)$, we use the chain rule and the Implicit Function Theorem (IFT) sequentially. First, applying the IFT to $g(a, w) = 0$ with respect to w:

$$\frac{\partial g}{\partial a}\frac{\partial a^*}{\partial w} + \frac{\partial g}{\partial w} = 0 \quad \Longrightarrow \quad \frac{\partial a^*}{\partial w} = -\frac{g_w}{g_a}$$

Second, applying the chain rule to relate $\frac{\partial a^*}{\partial w}$ to $\frac{\partial a^*}{\partial \vartheta}$:

$$\frac{\partial a^*}{\partial \vartheta} = \frac{\partial a^*}{\partial w}\frac{\partial w}{\partial \vartheta} = -\frac{g_w(a^*)}{g_a(a^*)}\frac{\partial w}{\partial \vartheta} \tag{17}$$

Step 2: Sign the denominator, g_a: The denominator $g_a = \frac{\partial g}{\partial a}$ is the second-order condition (SOC), f_{aa}, which is strictly negative due to risk aversion ($U'' < 0$):

$$g_a < 0$$

Step 3: Calculate and sign the cross-partial term, g_w: We calculate $g_w = \frac{\partial g}{\partial w}$ by differentiating the FOC with respect to the subjective weight w, holding a (and thus W_1, W_2) constant:

$$g(a, w) = w\, U'(W_1)(r_1 - r_f) + (1 - w)U'(W_2)(r_2 - r_f)$$

Differentiating yields:

$$g_w = U'(W_1)(r_1 - r_f) - U'(W_2)(r_2 - r_f)$$

We now sign g_w using the model assumptions ($r_1 > r_f > r_2$ and $U' > 0$):

- **Term 1 (Good State):** $U'(W_1)(r_1 - r_f)$ is $(+) \cdot (+) = (+)$.
- **Term 2 (Bad State):** $U'(W_2)(r_2 - r_f)$ is $(+) \cdot (-) = (-)$.

Since the positive term (Term 1) is followed by the subtraction of a negative term (Term 2), the result is strictly positive:

$$g_w = \underbrace{U'(W_1)(r_1 - r_f)}_{>0} - \underbrace{U'(W_2)(r_2 - r_f)}_{<0} > 0$$

Step 4: Sign the probability weighting term, $\frac{\partial w}{\partial \vartheta}$: The theorem specifies the condition that the parameter ϑ "overweights the good state", which is formally defined as:

$$\frac{\partial w}{\partial \vartheta} > 0$$

This condition means that a rise in ϑ increases the subjective perception of the probability of the good state, $w(p)$, relative to the bad state, $1 - w(p)$.

Step 5: Conclusion: Substitute the determined signs back into the IFT expression (17):

$$\frac{\partial a^*}{\partial \vartheta} = -\frac{g_w}{g_a}\frac{\partial w}{\partial \vartheta}$$

$$\frac{\partial a^*}{\partial \vartheta} = -\frac{(+)}{(-)}(+) = (-) \cdot (-) \cdot (+) = (+)$$

$$\implies \frac{\partial a^*}{\partial \vartheta} > 0$$

Thus, an increase in the parameter ϑ (which overweights the good state) unambiguously increases the optimal risky investment a^*. □

Corollary 2 (Rational benchmark)**.** *If the probability weighting is rational, i.e., $w(p) = p$ for all objective probabilities p (which implies $\frac{\partial w}{\partial \vartheta} = 0$), then $\frac{\partial a^*}{\partial \vartheta} = 0$, and Theorem 3 collapses to the standard expected-utility case.*

Proof. The corollary is a specialization of the comparative static result derived in Theorem 3 regarding the parameterization of the probability weighting function.

Step 1: Recall the general comparative static from Theorem 3: Theorem 3 establishes the relationship between the optimal risky share a^* and the probability weighting parameter ϑ using the implicit function theorem (IFT) and chain rule:

$$\frac{\partial a^*}{\partial \vartheta} = -\frac{g_w(a^*, w)}{g_a(a^*, w)}\frac{\partial w}{\partial \vartheta} \tag{18}$$

where $g(a, w) = 0$ is the FOC, g_a is the strictly negative SOC ($g_a < 0$), and g_w is the partial derivative of the FOC with respect to the subjective weight w:

$$g_w(a) = U'(W_1)(r_1 - r_f) - U'(W_2)(r_2 - r_f)$$

Step 2: Apply the rational benchmark condition: The condition for the rational benchmark is that the investor uses the objective probability p directly, meaning the subjective weighting function $w(p)$ is fixed to p:

$$w(p) = p$$

Since w is defined explicitly as equal to the objective probability p, it is constant with respect to any parameter ϑ that describes a distortion or weighting of p:

$$\frac{\partial w}{\partial \vartheta} = \frac{\partial p}{\partial \vartheta} = 0$$

Step 3: Conclude the effect on a^*: Substituting the result from Step 2 ($\frac{\partial w}{\partial \vartheta} = 0$) into the general comparative static expression (18):

$$\frac{\partial a^*}{\partial \vartheta} = -\frac{g_w(a^*, w)}{g_a(a^*, w)} \cdot 0$$

$$\implies \frac{\partial a^*}{\partial \vartheta} = 0$$

The conclusion is that when the probability weighting is rational, the optimal risky allocation a^* is invariant to the weighting parameter ϑ. This confirms that the model correctly reduces to the standard expected-utility framework when behavioral distortions are absent. □

2.3 Closed-form comparative statics for CRRA (local)

To make the wealth effect explicit, consider CRRA utility:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}, \qquad \gamma > 0,\ \gamma \neq 1.$$

where γ is coefficient of relative risk aversion which measures how strongly an investor dislikes risk relative to wealth.

Then $U'(W) = W^{-\gamma}$ and $U''(W) = -\gamma W^{-\gamma-1}$. Plugging into (10) yields:

$$w\, W_1^{-\gamma}(r_1 - r_f) + (1 - w)\, W_2^{-\gamma}(r_2 - r_f) = 0. \tag{19}$$

Using (6), W_1 and W_2 are affine in a. The derivative terms become:

$$g_a = -\gamma\Big[\, w\, W_1^{-\gamma-1}(r_1 - r_f)^2 + (1-w)\, W_2^{-\gamma-1}(r_2 - r_f)^2\Big] < 0, \tag{20}$$

$$g_\pi = \gamma A\Big[\, w\, W_1^{-\gamma-1}(r_1 - r_f) + (1-w)\, W_2^{-\gamma-1}(r_2 - r_f)\Big]. \tag{21}$$

Hence, from (14),

$$\frac{da^\star}{d\pi^e} = \frac{g_\pi}{g_a} = A\,\frac{\big[w\, W_1^{-\gamma-1}(r_1 - r_f) + (1-w)\, W_2^{-\gamma-1}(r_2 - r_f)\big]}{w\, W_1^{-\gamma-1}(r_1 - r_f)^2 + (1-w)\, W_2^{-\gamma-1}(r_2 - r_f)^2}. \tag{22}$$

Under the canonical ordering $r_1 > r_f > r_2$, the numerator in (22) is negative (the second term is negative because $r_2 - r_f < 0$), so $\frac{da^\star}{d\pi^e} < 0$. This recovers Theorem 2 and shows explicitly how wealth levels W_i scale the sensitivity through $W_i^{-\gamma-1}$.

Figure 1 provides a graphical counterpart to the analytical results derived above. The solid orange line and dashed blue line depict the opportunity sets corresponding to baseline and higher inflation expectations. Because inflation expectations π^e uniformly reduce real returns without altering the slope of excess returns $(r_1 - r_f)$ and $(r_2 - r_f)$, the opportunity set shifts downward in parallel when π^e rises. This movement represents a pure wealth effect that compresses the investor's attainable combinations of (W_1, W_2) across states.

The green indifference curves represent behavioral preferences under the baseline weighting function $w(p)$, while the blue curve represents an alternative scenario in which probability weighting places greater emphasis on the good state. The point of tangency between the baseline opportunity line and the yellow indifference curve identifies the optimal risky allocation $a^\star$. As inflation expectations increase, the tangency point shifts inward along the opportunity locus, confirming the analytical result in Theorem 2 that $\frac{da^\star}{d\pi^e} < 0$.

Comparatively, when the investor overweights the good state (higher $w(p)$), the indifference curves rotate outward, reflecting reduced perceived risk and higher marginal utility in the favorable outcome. This behavioral distortion increases the risky share, consistent with Theorem 3, where $\frac{\partial a^\star}{\partial \vartheta} > 0$. The 45-degree line, plotted as a reference, denotes the certainty locus $W_1 = W_2$. The feasible region below the opportunity frontier highlights all attainable wealth combinations given the investor's endowment and market structure. Together, these graphical features visually demonstrate how inflation expectations and probability weighting jointly determine optimal portfolio selection in the (W_1, W_2) space.

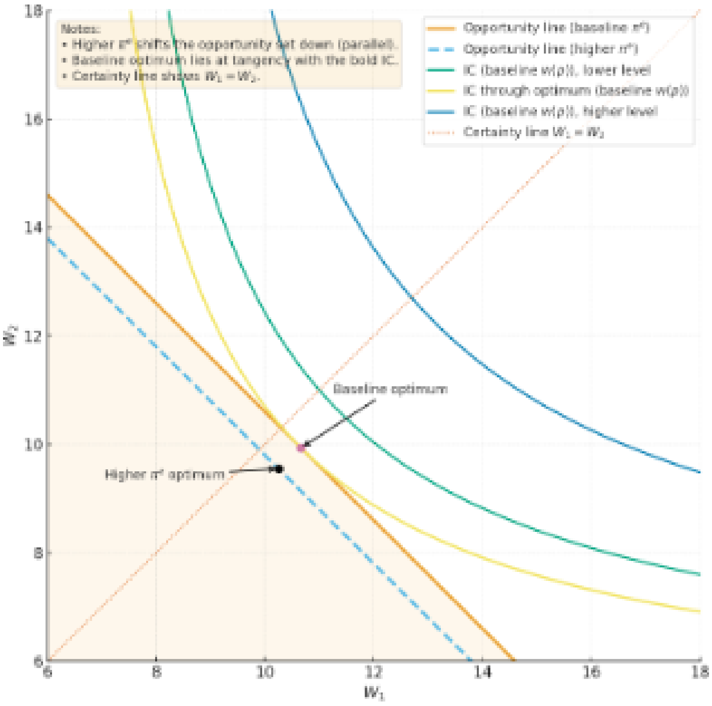

Figure 1: Opportunity line and indifference curves under shifts in inflation expectations π^e and probability weighting $w(p)$

Figure 2 visualizes the investor's expected utility $f(a; \pi^e, p)$ as a joint function of the risky investment share a and inflation expectations π^e. Each contour line traces combinations of (a, π^e) that yield the same level of utility under the model. Darker regions represent lower utility, whereas lighter regions indicate higher attainable utility given the investor's wealth, risk aversion, and behavioral weighting $w(p)$.

The bold solid curve corresponds to the locus of optimal allocations $a^\star(\pi^e)$, that is, the ridge line of maximal utility for each inflation expectation. This ridge illustrates that as π^e rises, the investor's real wealth declines, reducing the feasible opportunity set and shifting the optimal risky position leftward. Consequently, the optimal risky investment $a^\star$ declines monotonically with inflation expectations, consistent with Theorem 2, which establishes $\frac{da^\star}{d\pi^e} < 0$ under DARA preferences.

Economically, Figure 2 provides a local visualization of the wealth effect embedded in the comparative-statics analysis. The downward slope of the $a^\star(\pi^e)$ path reflects that higher expected inflation—by eroding real returns—induces investors to rebalance away from risky assets toward the risk-free component. This contour representation thus complements Figure 1 by translating the theoretical geometry of (W_1, W_2) space into the policy space (a, π^e), making explicit the continuous relationship between inflation expectations and optimal portfolio risk exposure.

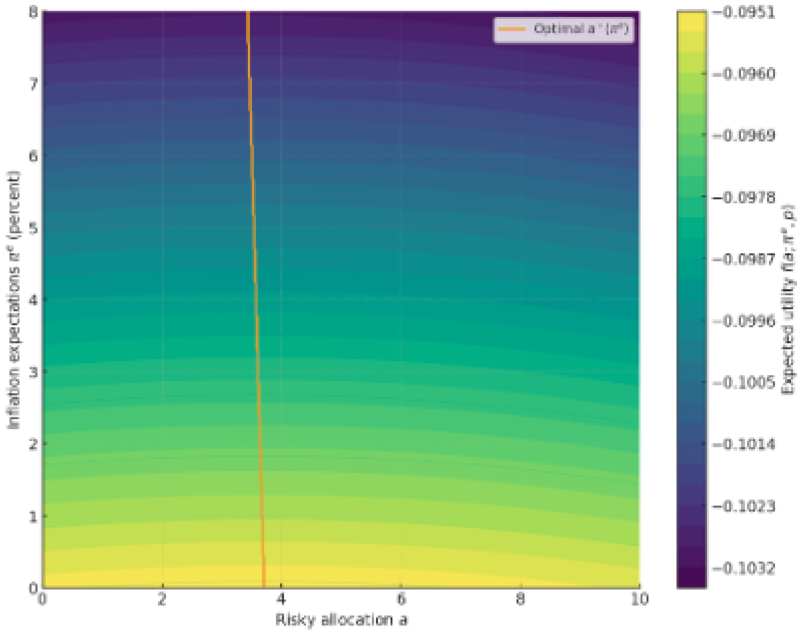

Figure 2: Contour map of expected utility $f(a;\pi^e,p)$ over the (a,π^e) space, with the optimal policy path $a^\star(\pi^e)$ indicated by the solid curve.

3 Conclusion and Policy Implications

This paper develops a unified behavioral–macro portfolio model that analytically links inflation expectations, probability weighting, and optimal risky investment under CRRA preferences. Within a single-period framework, we show that higher inflation expectations uniformly reduce real returns, generating a pure wealth effect that shifts the opportunity frontier downward without altering its slope. Under decreasing absolute risk aversion (DARA), this wealth compression lowers the investor's optimal risky share. In contrast, a higher subjective weight on the good state—as in a Prelec-type probability weighting—rotates indifference curves outward, raising the optimal risky exposure. Together, these results provide a closed-form behavioral foundation for how macroeconomic expectations and subjective probability distortions jointly shape portfolio allocation.

Our analytical results have several implications for monetary policy and financial stability. First, they highlight that persistent increases in inflation expectations may systematically reduce investors' risk appetite and shift wealth toward safer assets, amplifying risk-off episodes in financial markets. Central banks should therefore monitor not only the level of inflation expectations but also their dispersion across households and institutions, since heterogeneous expectations can generate portfolio imbalances and asset-price volatility. Second, the framework suggests that improving the credibility and transparency of monetary communication can stabilize investors' real-return expectations, mitigating behavioral distortions in risk-taking. Finally, the model's comparative statics provide a theoretical rationale for incorporating behavioral probability weighting into macroprudential stress testing, where distorted perceptions of "good" and "bad" states affect capital allocation and liquidity preference.

While the model yields clean analytical insights, several limitations merit attention. First, the single-period structure abstracts from dynamic feedback between expectations, wealth accumulation, and learning over time. Extending the framework into a multi-period or stochastic dynamic general equilibrium (DSGE) environment would allow analysis of persistence and intertemporal substitution effects. Second, our behavioral weighting function is parameterized exogenously; empirical calibration using experimental or survey-based probability judgments could better quantify the magnitude of distortion across investor types. Third, the model focuses on two discrete states of the world. Incorporating continuous return distributions or ambiguity-averse preferences (as in Epstein and Schneider, 2008) would enhance realism and robustness.

Future research may integrate this behavioral portfolio mechanism into asset-pricing and macro-finance models to study how inflation expectations propagate through financial markets, or how policy announcements dynamically reshape investors' subjective state probabilities. Such extensions could inform central-bank communication strategies and the design of behavioral macroprudential tools in inflationary environments.

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